1. EQUATIONS FOR HEAT TRANSFER IN BOILERS

1.1 Equations for heat transfer by convection

1.1.1 Natural convection on vertical plate and vertical cylinder[1]

The furnaces of fire-tube boilers are, in most cases, round and horizontal, in some boilers are also vertical. Based on experiments of different authors, the equation covering the turbulent and laminar flow in *Ra*=|Gr|.Pr=0.1 up to 10^{12} range applicable for vertical plane plates had been written:

$$Nu = \left\{0.825 + 0.387\left[Ra \cdot f_1(\mathrm{Pr})\right]^{1/6}\right\}^2$$

Equation 1

Function f_1(Pr) takes into account the influence of Prandtl-number in $0.001<\mathrm{Pr}<\infty$ range:

$$f_1(\mathrm{Pr}) = \left[1 + \left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{-9/16}$$

Equation 2

For so-called inflow length $l = \frac{A}{O}$, the height of the plate is used. Heat transfer calculated by the above equation is valid for intermediate flow types where the flow is turning from laminar to turbulent while the latter is not yet completely formed. The accuracy of the equation is sufficiently maintained over the entire range of Rayleigh-numbers. It is also applicable in cases when the temperature over the plate height is not uniform (acceptable inaccuracy of 4% becomes higher). This is accomplished by using a temperature at the 50% plate height. The above equation can be used as well for vertical cylinder in the form:

$$Nu = Nu_{pl} + 0.97\frac{h}{D}$$

Equation 3

1.1.2 Natural convection on horizontal cylinder

In case of horizontal cylinder its diameter *D* is used to calculate inflow length:

$$l = \frac{\pi}{2}D \text{ [m]}$$

Equation 4

---

[1] Covers also the area where natural and forced convection overlap.

Heat transfer by natural convection on horizontal cylinder is:

$$Nu = \left\{0.75 + 0.387\left[Ra \cdot f_3(\mathrm{Pr})\right]^{1/6}\right\}^2$$

Equation 5

Function $f_3(\mathrm{Pr})$ considers influence of Prandtl-number over entire range $0<\mathrm{Pr}<\infty$:

$$f_3(\mathrm{Pr}) = \left[1 + \left(\frac{0.559}{\mathrm{Pr}}\right)^{9/16}\right]^{-16/9}$$

Equation 6

The material and media properties are calculated at mean logarithmic temperature, t_m, as the result of the mean logarithmic temperature of media, t_f, (see Figure 19) and mean wall temperature, t_w:

$$t_m = \frac{1}{2}(t_f + t_w) \text{ [°C]}$$

Equation 7

1.1.3 Forced convection on plane plate

1.1.3.1 Laminar flow

The Nusselt-number for the plate of length *l*:

$$Nu_{lam} = 0.664\sqrt{\mathrm{Re}}\sqrt[3]{\mathrm{Pr}}$$

Equation 8

Reynolds number limits validity range of above equation

$$\mathrm{Re} = \frac{wl}{\upsilon} < 10^5$$

Equation 9

and also by Prandtl-numbers from 0.6 up to 2000. The coefficient 0.664 in the above equation increases with an increase of the Prandtl-number and reaches a value of 0.703 at Pr=1000 (all the material properties are arithmetic averages between the inlet and exit temperatures). The formation of a laminar boundary layer is guaranteed by properly shaping the plate brims. At the distance, x_{crit}, from plate brim being defined by critical Reynolds-number the laminar boundary layer turns into turbulent:

$$\mathrm{Re}_{x,crit} = w \cdot x_{crit} / \upsilon$$

Equation 10

The critical Reynolds-number, which depends on surface harshness and turbulence grade, is assumed to be $5 \cdot 10^5$ (the more turbulent the flow the higher the critical number and vice versa).

1.1.3.2 Turbulent flow

In the case of a turbulent boundary layer, the average Nusselt-number in the $5\cdot 10^5<Re<10^7$ and Pr=0.6-2000 ranges is computed by following equation (the material properties are computed at mean arithmetic temperature):

$$Nu_{turb} = \frac{0.037\,\mathrm{Re}^{0.8}\,\mathrm{Pr}}{1+2.443\,\mathrm{Re}^{-0.1}(\mathrm{Pr}^{2/3}-1)}$$

Equation 11

If there is an overhanging brim then there is a turbulent boundary layer in front of it. In such a case, the following equation is used:

$$Nu = \frac{\xi/8\,\mathrm{Re}\,\mathrm{Pr}}{1+12.7\sqrt{\xi/8}(\mathrm{Pr}^{2/3}-1)}$$

Equation 12

Average plate-resistance coefficient for the turbulent flow valid up to $Re=10^7$:

$$\xi/8 = 0.037\,\mathrm{Re}^{-0.2}$$

Equation 13

In most cases, due to blunt plate brims and turbulence of the flow, the laminar boundary layer is not formed over the entire surface of the plate. Due to this, the so-called average balance curve, which takes into account also a turbulent boundary layer in Pr=0.6-2000 and $10<Re<10^7$ range, is applied:

$$Nu_{1,0} = \sqrt{Nu_{lam}^2 + Nu_{turb}^2}$$

Equation 14

A heat-flow direction (heating or cooling) must be considered since the temperature dependency on the thermal material properties has an influence on heat transfer in the case of turbulent flow. In the case of cooling of the gas, this correction is not necessary as the Prandtl-number is almost temperature independent.

1.1.4 Forced convection in tubes and channels

For $Re<2300$, the flow is always laminar. It turns into a turbulent one while when Reynolds number becomes higher then 2300 but turbulence is fully formed much later (at $Re>10^4$). Thus, in the $2300<Re<10^4$ range, the flow is intermediate, which means the shape of tube-brim and type of inflow must be taken into account. The equations are valid for cases with constant surface temperatures, sharp edges without rounding, and for circular tubes (for other forms of cross-sections an equivalent diameter is used).

1.1.4.1 Laminar flow

For gases and liquids in tubes of inflow lengths, *l,* the average Nusselt-number valid in *Re*Pr d/l=0.1 up to 10^4 range is computed by equation:

$$Nu_0 = \left[3.65 + \frac{0.19(\mathrm{Re}\,\mathrm{Pr}\,d/l)^{0.8}}{1 + 0.117(\mathrm{Re}\,\mathrm{Pr}\,d/l)^{0.467}} \right]$$

Equation 15

To compute the material properties, the mean arithmetic temperature is used. In cases of short tube lengths ($\frac{d}{l} > 0.1$), the equation above is shorter:

$$Nu_0 = 0.644 \sqrt[3]{\mathrm{Re}\frac{d}{l}}$$

Equation 16

For the heat transfer at short intake lengths ($\frac{d}{l} > 0.1$), the one of above equations is used, which gives yields Nu-number. Heat-flow direction (heating or cooling) in the case of gas, has no influence on heat transfer.

1.1.4.2 Turbulent flow

In the case of turbulent flow of liquids, the equation below covers also an intermediate area and is valid in the *Re*=2300-10^7 range:

$$Nu_0 = \frac{\xi/8(\mathrm{Re}-1000)\,\mathrm{Pr}}{1 + 12.7\sqrt{\xi/8}(\mathrm{Pr}^{2/3}-1)}\left[1 + \left(\frac{d}{l}\right)^{2/3}\right]$$

Equation 17

The pressure-loss factor is determined by:

$$\xi = (1.82\log_{10}\mathrm{Re} - 1.64)^{-2}$$

Equation 18

In the case of short tubes (0.1<$\frac{d}{l}$<1), the one of above equations is used in 2300<*Re*<10^4 range, which gives larger Nu-number. Material properties are computed at mean arithmetic temperatures. In the case of gas flow, the exponent in brackets of Equation 17 equals 0, which means that the heat-flow direction has no influence on heat transfer.

1.1.5 Forced convection in ring-slot

Figure 1 shows three cases of heat transfer in ring-slot. The tubes are assumed concentric and equivalent diameter of ring-slot is:

$$d_h = d_a - d_i \text{ [m]}$$

Equation 19

1.1.5.1 Laminar flow

In the case of a fully formed laminar flow of gases and liquids, the Nusselt-number is determined by equation:

$$Nu = \left[Nu_\infty + f\left(\frac{d_i}{d_a}\right) \frac{0.19\left(\mathrm{Re}\,\mathrm{Pr}\frac{d_h}{l}\right)^{0.8}}{1 + 0.117\left(\mathrm{Re}\,\mathrm{Pr}\frac{d_h}{l}\right)^{0.467}} \right] \cdot \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_w}\right)^{0.11}$$

Equation 20

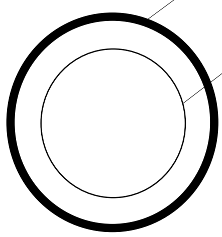

Figure 1: Ring-slot

The factors Nu_∞ and $f\left(\frac{d_i}{d_a}\right)$ in the above equation are different for each case and for the inflow-length, *l,* a length of the ring-slot is used. For heat transfer to the exterior tube, as is the case when the hot tube is inserted in furnace of boiler, the following expression is used:

$$Nu_\infty = 3.66 + 1.2\left(\frac{d_i}{d_a}\right)^{-0.8}$$

Equation 21

$$f\left(\frac{d_i}{d_a}\right) = 1 + 0.14\sqrt{\frac{d_a}{d_i}}$$

Equation 22

1.1.5.2 Turbulent flow

For turbulent flow, the modified equations of turbulent flow in tubes and channels are be used:

$$\frac{Nu}{Nu_0} = f\left(\frac{d_i}{d_a}\right)$$

Equation 23

Heat transfer to the exterior tube:

$$\frac{Nu}{Nu_0} = 0.86\left(\frac{d_a}{d_i}\right)^{0.16}$$

Equation 24

Index, 0, in the above equations stands for Nu-number for flow in tubes obtained from equations for flow in tubes.

The material properties are calculated at the mean logarithmic temperature of coolant (boiler water), t_m, according to Figure 19 as a result of the temperature pairs, t_1, (inlet) and, t_2, (exit):

$$t_m = \frac{1}{2}(t_1 + t_2) \text{ [°C]}$$

Equation 25

1.2 Equations for heat transfer to boiling water

In fire-tube steam boilers, the following equation [3] can be used in 0-100 bar range for the heat transfer to boiling water:

$$\alpha = 3.489 \cdot q^{0.7} \cdot (p \cdot 0.98692)^{0.15} \text{ [W/m}^2\text{K]}$$

Equation 26

1.3 Equations for heat transfer by gas radiation

Gases are capable, as are solid bodies, to absorb and emit energy by radiation. One- and two-atomic gases (e.g. nitrogen, oxygen, and hydrogen) radiate in narrow wavelength bands and are practically totally transparent to radiation. Heavier gases, as carbon dioxide, water vapor, sulfur dioxide, ammonia, and others possess an extensive absorptive and radiation capability. Of primary importance for combustion are carbon dioxide CO_2, water vapor H_2O, and sulfur dioxide SO_2 since they are generated during the combustion. The SO_2 can be neglected due to its negligible partial pressure in the flue-gases (in cases of gas combustion, the SO_2 is not generated at all).

For H_2O compared to CO_2 the influence of a partial pressure is stronger than the influence of a gas layer thickness (mean beam length). Total radiation of both components is a little bit smaller than the calculated sum of both components separately due to partial overlapping of emission and absorption bands. This means that a part of the emitted energy from CO_2 is absorbed by H_2O and vice versa. As this only happens noticeably when the gas layer thickness exceeds 4m, the calculation error remains within 2-4%. This error is, in the case of fire-tube boilers, negligible because the diameter of the furnace of the fire-tube boilers has never reached 4m.

The equations for gas radiation are valid for cases when gas layer thickness is equal in all directions, which does not occur in practice. Therefore, the equivalent gas layer thickness, a mean beam length, must be calculated from the gas volume and area of gas surrounding surfaces [3]:

$$s = 0.9\frac{4V}{A} = 3.6\frac{V}{A} \text{ [m]}$$

Equation 27

1.3.1 Carbon dioxide radiation

Based on experiments, Alfred Schack [30] developed an equation, which delivers the energy, radiated from CO_2 gas with an inaccuracy within ±4%. For a black body of 0 K, temperature the equation follows:

$$q_{CO_2} = 1.163 \cdot \left\{ K_1\left(1 + 0.026\frac{t}{100}\right)\varphi\left(18ps\sqrt{\frac{273}{T}}\right) + K_2\left(1 + 0.031\frac{t}{100}\right) \cdot \left[1 - \frac{0.6}{115ps\sqrt{\frac{273}{T}}} \cdot \left(1 - e^{-115ps\sqrt{\frac{273}{T}}}\right) - \frac{0.4}{ps\left(140 + \frac{650}{\frac{T}{1000}} - 115\sqrt{\frac{273}{T}}\right)} \cdot \left(e^{-115}ps\frac{273}{T} - e^{-\left(140 + \frac{650}{\frac{T}{1000}}\right)^{ps}}\right)\right] + K_3\varphi(32ps)\right\} \text{ [W/m}^2\text{]}$$

Equation 28

Factor φ in above equation is an abbreviation:

$$\varphi(x) = 1 - \frac{1 - e^{-x}}{x}$$

Equation 29

The above abbreviation takes in 18*ps* - case the following form:

$$\varphi\left(18ps\sqrt{\frac{273}{T}}\right) = 1 - \frac{1 - e^{-18ps\sqrt{\frac{273}{T}}}}{18ps\sqrt{\frac{273}{T}}}$$

Equation 30

and in 32*ps* - case the form:

$$\varphi(32ps) = 1 - \frac{1 - e^{-32ps}}{32ps}$$

Equation 31

Factors K_1, K_2 and K_3 in Equation 28 represent the radiation of an indefinitely long layer of CO_2 in corresponding absorption bands [30, Table 1] as calculated by Planck's radiation law with dependence on temperature, T, which is MRT in this case.

As there are no black bodies of 0 K temperature in nature, the Equation 28 is transformed in order to take into account actual conditions:

$$q_{CO_2} = \varepsilon\left[q_{CO_2,T} - \left(\frac{T}{T_w}\right)^{0.65} \cdot q_{CO_2,T_w}\right] \text{ [W/m}^2\text{]}$$

Equation 32

Index, *T*, in the above equation stands for radiation of CO_2 at MRT and index, *Tw*, stands for radiation of CO_2 at mean wall temperature.

1.3.2 Water vapor radiation

Similar to CO_2 the equation for H_2O radiation was developed [30]. Its validity range is from *ps*=0-0.36barm and *t*=400-1900°C, which is sufficient for application in the fire-tube boilers:

$$q_{H_2O} = 1.163 \cdot (40 - 73ps)(ps)^{0.6}\left(\frac{T}{100}\right)^{2.32 + 1.37\sqrt[3]{ps}} \text{ [W/m}^2\text{]}$$

Equation 33

For gas temperatures over 650°C, theoretically, the correction factor needs to be applied. In normal conditions, as this in the case with fire-tube boilers, the coefficient can be omitted without noticeable error.

As analog to the CO_2 case for real conditions, the above equation becomes:

$$q_{H_2O} = \varepsilon\left(q_{H_2O,T} - q_{H_2O,T_w}\right) \text{ [W/m}^2\text{]}$$

Equation 34

Index, *T*, in the above equation stands for radiation of H_2O at MRT, and index, *Tw*, for radiation of H_2O at mean wall temperature.

As the walls surrounding the radiating gas emit a part of the energy received by the gas radiation back to the gas, the actual surface emissivity of walls is higher. According to [3], in an emissivity range of 0.8-1.0, the following approximation is allowed:

$$\varepsilon_{w,actual} = \left(\frac{\varepsilon_w + 1}{2}\right)$$

Equation 35

Table 1: Radiation of an indefinitely long layer of carbon dioxide

temperature °C	K_1 (band 1) (W/m^2) λ_{max}=2.7μm $\Delta\lambda$=0.21μm	K_2 (band 2) (W/ m^2) λ_{max}=4.3μm $\Delta\lambda$=0.3μm	K_3 (band 3) (W/ m^2) λ_{max} =15.9μm $\Delta\lambda$=4.6μm
200	6	80	280
300	75	279	430
400	225	645	610
500	630	1240	760
600	1440	2040	970
700	2850	3060	1150
800	4800	4210	1350
900	7200	5590	1510
1000	10200	7020	1720
1100	13900	8540	1910
1200	18600	10400	2110
1300	23500	12200	2300
1400	28900	14200	2500
1500	35000	16200	2690
1600	41400	18100	2890
1700	48100	20200	30600
1800	55400	22200	3270
1900	63000	24400	3480
2000	70600	26700	3690
2100	79400	29000	3860
2200	88000	31400	4060